Fractions We Can't Ignore:

The Nonsymbolic Ratio Congruity Effect

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The symbolic representation of whole numbers is a uniquely human cultural achievement. Although the use of these symbols themselves represents a transformative recycling of human cognitive architecture (Dehaene & Cohen, 2007), they acquire even greater power when they are combined to represent new classes of numbers like fractions (Siegler, Thompson, & Schneider, 2011). Indeed, recent research has drawn attention to the pivotal role fractions knowledge plays in human numerical development (Fazio, Bailey, Thompson, & Siegler, 2014; Siegler, Fazio, Bailey, & Zhou, 2013). They eventually come to represent complex concepts such as ratio, rate, and probability, providing the foundation for higher mathematics, science and associated technologies that are hallmarks of modern society. Despite the importance of fractions for mathematical cognition, the cognitive foundations of fraction understanding remain elusive.

When faced with the question of how a relatively ancient human brain structure enables facility with symbolic mathematics – a recent invention on the evolutionary scale – some have sought to identify core competencies upon which symbolic number concepts might rest. Most of these arguments share a general orientation with Deheane and Cohen’s (2007) neuronal recycling hypothesis: More recent cultural inventions such as mathematics and reading co-opt pre-existing cognitive architectures to support new competencies. In the case of number, the most frequently cited core competencies are those supporting the enumeration of discrete sets, such as the approximate number system (ANS) and the object tracking system (Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010). These competencies are widely thought to support the acquisition of whole numbers via specialized learning mechanisms (Feigenson et al., 2004; Piazza, 2010). Other classes of numbers – specifically fractions and irrational numbers – are thought by some to go beyond the constraints of these core architectures.[1] On this prevailing view, fractions are a product of human artifice and are therefore not naturally compatible with pre-existing cognitive architectures. However, several lines of research have indicated that fractions may not be as artificial as innate constraint theorists have argued (e.g., Jacob, Vallentin, & Nieder, 2012; Matthews & Chesney, 2015; McCrink, Spelke, Dehaene, & Pica, 2013; McCrink & Wynn, 2007).

The current research provides evidence in support of an expanded view of mathematical cognition which posits that learning about symbolic fractions is impacted by architectures that process the magnitudes of nonsymbolic ratios (Lewis, Matthews, & Hubbard, 2015). We reveal a nonsymbolic Ratio Congruity Effect (RCE) that demonstrates automatic processing of irrelevant nonsymbolic ratio magnitudes during symbolic fractions comparisons. This effect operates autonomously alongside previously identified symbolic numerical distance effects and size congruity effects for absolute size. The key contribution is to demonstrate that perceptually-based processing of nonsymbolic magnitude interfaces with the processing of symbolic fractions.

Background

Recent studies have brought new attention to the processes underlying the representation of fractions magnitudes (e.g., DeWolf, Grounds, Bassok, & Holyoak, 2014; Fazio et al., 2014; Jacob et al., 2012; Kallai & Tzelgov, 2009; Meert, Grégoire, & Noël, 2010). It is frequently noted that, unlike the case with whole numbers, even educated adults show considerable difficulties learning about and processing symbolic fractions (e.g., DeWolf et al., 2014). Several researchers have cited these pervasive difficulties with fractions as evidence for a fundamental incompatibility between core human cognitive architectures and fraction understanding. Feigenson et al. (2004) typified this innate constraints account, arguing that fractions are difficult

[1] Indeed, recent research suggests that very large whole numbers also tax the constraints of these systems (e.g., Landy, Silbert, & Goldin, 2013).

because they are far removed from the intuitions provided by core systems (see also Gelman & Williams, 1998[2]). As a result of this prevailing belief, most cognitive psychological theories of numerical development have focused on the acquisition of whole numbers, relegating fractions to a secondary status (Siegler et al., 2011).

Despite the widely held belief that fractions are in some sense artificial, a growing body of evidence suggests humans and non-human primates possess neurocognitive architectures that support the representation and processing of nonsymbolic ratios. For instance, Vallentin and Nieder (2008) trained adult humans and monkeys on match-to-sample tasks using ratios formed by pairs of lines (Fig. 1a). Humans and monkeys performed far better than chance, showing considerable sensitivity to ratio magnitudes. Moreover, single-cell recordings from monkeys revealed individual neurons that responded preferentially to specific nonsymbolic ratios.

Insert Fig. 1 about here

Much other work has revealed sensitivity to nonsymbolic ratios among a wide range of subject populations, including pre-verbal infants (Duffy, Huttenlocher, & Levine, 2005; McCrink & Wynn, 2007), school-aged children (Boyer & Levine, 2012; Sophian, 2000; Spinillo & Bryant, 1991), and typically developing adults (Hollands & Dyre, 2000; Jacob & Nieder, 2009). Indeed, Matthews and Chesney (2015) showed that adults accurately compared nonsymbolic ratios even when compared across different formats (e.g., ratios of dot arrays vs. ratios of circle areas, Fig.

[2] Despite published statements by Rochelle Gelman and Stanislas Dehaene suggesting fractions are not compatible with basic human architecture, we note that in other work, they make hypotheses to the contrary suggesting that basic number modules are compatible with rational numbers (McCrink, Spelke, Dehaene, & Pica, 2013) or even real numbers (Gallistel & Gelman, 2000). These points notwithstanding, their published arguments about innate constraints of the system are cited very frequently and continue to exert considerable sway.

1b). The ability to compare ratios despite their instantiations in different formats suggests that sensitivity to nonsymbolic ratio magnitudes is abstract on some level. No studies to date, however, have assessed the degree to which nonsymbolic ratio processing automatically interfaces with symbolic processes. We investigated the potential interaction of nonsymbolic and symbolic architectures using a Stroop-like paradigm that tested for nonsymbolic RCEs during the processing of symbolic fractions.

The Nonsymbolic Ratio Congruity Effect

Human processing of whole number symbols appears to be highly integrated with primitive cognitive architectures that process not only discrete numerosities (Piazza, Pinel, Le Bihan, & Dehaene, 2007) but also other nonsymbolic magnitudes such as size and even luminance (Cohen Kadosh & Henik, 2006; Henik & Tzelgov, 1982; Walsh, 2003). The interaction of symbolic whole number processing with physical magnitude has been revealed by the numerical size congruity effect (SiCE). The SiCE is a Stroop-like phenomenon in which the automatic processing of nonsymbolic magnitudes influences the intentional processing of number symbols and vice versa (e.g., Henik & Tzelgov, 1982). For example, when participants are asked to compare the numerical sizes of two symbolic digits, incongruent pairings of physical size (e.g., 2 vs. 4) interfere with performance, leading to slower and less accurate performance compared to congruent pairings (e.g., 2 vs. 4).

Recently, Kallai and Tzelgov (2009) investigated whether systems processing absolute physical size might also interact with systems that process symbolic fractions. In one experiment (Experiment 4), they investigated whether automatic processing of physical size congruence would affect intentional comparisons of symbolic fractions. They found SiCEs for comparing pairs of whole numbers, but not for pairs of fractions and concluded that physical magnitude does not automatically interact with fraction magnitude judgments. However, this failure to find SiCEs for fraction comparisons might also have reflected the type of nonsymbolic magnitude that was manipulated. Kallai and Tzelgov manipulated the *absolute* physical magnitudes (overall size) of the fractions being compared by manipulating the total area taken up by a given fraction. Fraction magnitude, however, is defined *relationally*; it is determined by the relative sizes of the components that comprise fractions as opposed to their absolute sizes. Therefore, we predicted that investigating relationally defined nonsymbolic ratio magnitudes might reveal integration of symbolic fraction processing with more primitive magnitude processing architectures.

Experiment 1

We tested this hypothesis by investigating whether nonsymbolic ratio magnitude automatically influences the speed and accuracy of symbolic fraction comparisons. Expanding upon previous work that has investigated SiCEs based on the *absolute sizes* of stimuli, we investigated RCEs that might emerge from *relative* magnitudes. Participants selected the larger of two symbolic fractions varying along three dimensions: the numerical values of the symbolic fractions compared (the relevant dimension), the congruity of the absolute physical sizes of the fractions relative to the symbolic fraction decision, and the congruity of the nonsymbolic ratios formed by the fonts used to print the numerators and denominators of each fraction.

Evidence of RCEs would have two implications for theories concerning processing of fraction magnitudes. First, RCEs for symbolic comparison tasks would demonstrate a heretofore unobserved relationship between two very different representations of ratios (i.e., symbolic and nonsymbolic). Second, because the nonsymbolic ratio manipulation is unfamiliar to participants, there would be little argument that any observed effects were the results of practice. They would instead be consistent with the existence of cognitive architectures specifically dedicated to processing the holistic magnitudes of non-symbolic ratios as suggested by Jacob et al. (2012).

Method

Participants

Participants were 40 undergraduates (35 female, M_{age} = 20.03) from a large Midwestern university participating for course credit.

Stimuli

The symbolic fractions used were irreducible proper fractions with single digit numerators and denominators, excluding ½ to avoid effects associated with its special status (see Schneider & Siegler, 2010). From these, we produced 86 fraction pairs with no shared components (i.e., no digit appeared in both fractions of a pair). Pairs were constructed this way to reduce reliance on componential strategies that might bypass the processing of holistic fraction magnitude (Meert, et al., 2010). The Appendix lists all fraction pairs used.

The numerator and denominator of each fraction were printed in different sized fonts, operationalized as the area of the implicit bounding box around each numeral (Fig. 2). The bounding box was operationalized as the font rectangle used by the graphics package in R. We adjusted the 'character expansion factor' of each numeral to make characters fill the box as completely as possible. This minimized any discrepancies between the bounding box as listed by the program and one that could be fit as tightly as possible if hand drawn around a given digit. Note that the "1" in Fig. 3 has a wide base and a serif, so that its bounding box is similar to that of other numbers drawn in the same font.

Insert Fig. 2 about here

Font sizes were systematically combined to produce different *absolute physical* *magnitudes* and *nonsymbolic font ratio*s. The absolute physical magnitude of a fraction was defined as the summed areas of the numerator and denominator fonts. This dimension is not defined relationally, as it is simply the summed area of fraction components. In contrast, the nonsymbolic font ratio was defined as the number generated by dividing the numerator font area by the denominator font area. Numerator font size was always smaller than denominator font size. Because this was defined relationally, it was possible to have small font ratios with large physical magnitudes and to have large font ratios with small absolute physical magnitudes (Fig. 2).

Fraction font sizes were determined for each trial with these restrictions:

1) Fonts of individual characters ranged in size from 17 x 24 pixels to 132 x 185 pixels;
2) Nonsymbolic font ratios composed of these characters ranged in value from approximately .1 to .9;
3) The absolute physical magnitude for a fraction was allowed to range from 12233 to 37800 sq pixels;
4) The ratio between the larger font ratio and smaller font ratio was always approximately 2:1. Note that this ratio is actually a *ratio of ratios* (see Fig. 1);
5) The ratio of the larger absolute physical magnitude to smaller absolute physical magnitude was always approximately 2:1.

For each trial, the program first randomly chose the value of the larger font ratio (between .2 and .9). The value of the smaller ratio was set to ½ of this value (e.g., .10 to .45). Next the larger absolute magnitude was randomly chosen from the range described above, and the smaller absolute magnitude was set to ½ of this value. This combination of font ratio and absolute magnitude completely defined the actual font sizes for each of the components used. This method allowed absolute physical magnitudes and nonsymbolic font ratio magnitudes to vary randomly across trials while constraining both to be approximately 2:1 within each comparison pair. These ratios were very close, but inexact approximations due to the necessity of rendering fonts in a discrete number of pixels. The larger fraction was on the right for half of the trials and on the left for the other half. This was also true for font ratio and for absolute physical magnitude.

Nonsymbolic font ratio was defined as *congruent* with the symbolic fraction decision when the smaller valued symbolic fraction was printed in the smaller nonsymbolic font ratio. Alternatively, it was defined as *incongruent* when the larger symbolic fraction was printed in the smaller font ratio. Likewise, absolute physical magnitude was considered congruent with the symbolic fraction decision when the smaller valued symbolic fraction was printed in the smaller absolute physical magnitude and incongruent when the larger symbolic fraction was printed in the smaller absolute physical magnitude (Fig. 3).

Insert Fig. 3 about here

Design

We manipulated font ratio congruity (congruent, incongruent) and absolute physical magnitude congruity (congruent, incongruent) within subjects. We manipulated the location of the larger symbolic fraction values (left, right) between subjects by randomly assigning participants to two separate test lists. Each test list contained all four within subject factor permutations for each fraction pair, but only contained one permutation of location for each fraction (e.g., one list contained four versions of $^{2}/_{9}$ vs. $^{3}/_{7}$, and the other contained four versions of $^{3}/_{7}$ vs. $^{2}/_{9}$). The absolute distance between the values of the symbolic fractions in each pair varied as did the absolute distances between the numerators in each pair and the denominators in each pair. The presentation order was randomized for each participant.

Procedure

Participants were instructed to select the larger fraction while ignoring differences in font size. Stimuli were presented on an LCD monitor (54.61 cm, measured diagonally) using a script written in PEBL (Mueller, 2012). Participants pressed “j” when they judged the right symbolic fraction to be larger and “f” when they judged the left to be larger. On each trial, participants first saw a fixation cross for 1000ms followed by a fraction pair. Stimuli remained on-screen until participants made a choice. The next trial commenced immediately following a response. Participants completed 5 practice trials followed by 344 experimental trials (86 pairs x 2 absolute physical congruity x 2 font ratio congruity).

Results

We used linear mixed models (LMM) to account for within-subject correlation among trials. We conducted separate analyses with RT (a linear model) and accuracy (a logistic model) as dependent variables to parallel analyses in prior literature (Schneider & Siegler, 2010). Models initially included two nonsymbolic independent variables (font ratio congruity and absolute magnitude congruity) and three symbolic ones: holistic distance (i.e. $|Fraction_1 – Fraction_2|$), numerator distance (i.e., $|Numerator_1 – Numerator_2|$), and denominator distance (i.e., $|Denominator_1 – Denominator_2|$). Preliminary analyses also indicated that fraction pairs in which the smaller symbolic fraction contained both a larger numerator and a larger denominator than the larger symbolic fraction (i.e., $^2/_9$ vs. $^1/_3$, $^5/_9$ vs. $^2/_3$, etc.) were responded to more slowly and less accurately than others (see also Ischebeck, Weilharter, & Körner, 2015). Consequently, we added a double symbolic incongruity factor to the LMM models to ensure that estimates were not unduly influenced by these items. This wholly symbolic factor was defined as incongruent when both the numerator and the denominator values of the smaller fraction were larger than the numerator and denominator of the larger fraction and as congruent otherwise. We also initially included several interaction terms: holistic distance × font ratio congruity, holistic distance × absolute physical magnitude congruity. holistic distance × double symbolic incongruity, absolute physical magnitude congruity × font ratio congruity, and absolute physical magnitude congruity × double symbolic incongruity. No interactions were significant, so we dropped the terms and reran the analyses confined to main effects.

We estimated fixed effects for all variables with random intercepts. Outliers were culled at the participant level; responses more than 3 standard deviations from a participant's mean RT were trimmed. This affected less than 2% of the data. As is standard in the literature, RT analysis included only error-free trials. However, because participants were very accurate overall (M = 93.4), RT analyses still included over 90% of all data points. Results from LMM analyses are presented in Tables 1a and 1b and in Fig. 4.[3]

Insert Tables 1a and 1b about here

Insert Fig. 4 about here

Analyses revealed the predicted font-based RCEs. Even though nonsymbolic font ratio was an irrelevant task dimension, participants were slower (M = 99.35 ms, $p < .01$) and less likely to be accurate (OR = .54, $p < .01$) when fractions were printed in font ratios that were

[3] We conducted analysis using raw RTs because it yielded results with easily interpretable units. However, RTs for comparisons tend to be positively skewed, violating normality assumptions, so we also analyzed log transformed RT data. Analyses of log RTs were compatible with those from raw scores, with the same variables emerging as significant and in the same direction across both analyses.

incongruent with the symbolic comparison decision (see Fig. 2). There were also congruity effects for absolute physical magnitude, contrary to the findings of Kallai and Tzelgov (2009). Participants were slower (M = 35.50 ms, p = .04) and less accurate (OR = .57, p < .01) when the overall physical sizes of a fraction pair were incongruent with the symbolic comparison dimension. Results further revealed a double symbolic incongruity effect: participants were considerably slower (M = 507.67 ms, p < .01) and less accurate (OR = .15, p < .01) when the symbolic fraction with the smaller holistic value contained both a larger valued numerator and a larger valued denominator than the larger symbolic fraction.

In addition to these congruity effects, results revealed multiple symbolic distance effects. First, participants exhibited distance effects based on holistic magnitude; they were slower and less accurate when the distance between the holistic values of two fractions was smaller. These distance effects based on holistic fraction magnitudes were the largest effects we found, which was to be expected given that fraction magnitude was the relevant dimension for comparison. Second, participants exhibited distance effects for RT due to componential processing as well. Participants were slower to respond when the numerator and denominator distances were smaller. There was also an insignificant trend toward participants being less accurate as denominator distance increased, consistent with some small whole number bias (e.g., Kallai &Tzelgov, 2009). However, distance between numerator components did not similarly affect accuracy. The findings of both holistic and componentially based distance effects were consistent with findings from prior literature (DeWolf et al., 2014; Kallai & Tzelgov, 2009; Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013).

Discussion

These results indicated that automatic processing of nonsymbolic ratios interfaces with processing symbolic fractions values. Elements of the design largely precluded the possibility that these results were artifacts of the fact that larger ratios were composed of digits that were closer in physical size. For example, in the comparison $\frac{4}{7}$ vs. $\frac{2}{5}$, the '4 and the '7' are more homogenous than the '2' and the '5', which might thereby render $\frac{4}{7}$ easier to read. Importantly, each comparison was presented in 8 total configurations such that sometimes $^{2}/_{5}$ was printed in the larger font ratio and sometimes $^{4}/_{7}$ was. In this way, homogeneity of component size was balanced across all configurations. Participants were only slower when the font ratio was incongruent, which rules out effects of readability due to homogeneity of components. Thus, the observed RCEs really do seem to be about the ratios comprised by printing components in different font sizes.

Our findings of an SiCE whereby physical size impacted symbolic fractions comparisons was counter to those of Kallai and Tzelgov (2009) in some respects. As with the current experiment, their Experiment 4 employed physical size as the irrelevant dimension for fraction comparisons, but several aspects of the experimental design may have attenuated size congruence effects for fractions relative to our protocol. First, Kallai and Tzelgov only used the integers 2, 3, and 4, and the fractions ½, ⅓, and ¼. This means all fractions used were unit fractions, which other research has shown fail to induce holistic processing (Bonato, Fabbri, Umiltà, & Zorzi, 2007; Ischebeck et al., 2015; Obersteiner et al., 2013). Moreover, it may have been that the combination of integers and fraction magnitudes in the same block altered participant approaches to fraction comparisons. We conducted Experiment 2 using a design tailored to find SiCEs to replicate these effects.

Experiment 2

In a relatively frequently cited work, Kallai and Tzelgov (2009) point to the lack of an

SiCE (i.e., that comparison of symbolic fractions was not affected by physical size) to conclude that "unlike natural numbers, fractions are not associated with specific size or quantity even in the [numerical comparison] task, where participants were asked to infer the numerical value of the fraction" (p. 1859). The lack of an SiCE was presumed in part to be due to fractions being encoded as generalized magnitudes less than one. However, the findings of Experiment 1 only make sense if symbolic fraction magnitudes are more differentiated. We felt it important to corroborate these contrary findings. A conservative reader might charge that our Experiment 1 had features that could possibly compromise the validity of the SiCE finding for absolute physical magnitude. Specifically, it is possible that the manipulation of font ratios – a novel stimulus for participants – prompted wholesale changes in fractions processing strategies and that the overall SiCE emerged as a result. We conducted Experiment 2 as a modification of Experiment 4 of Kallai and Tzelgov with a new sample of participants in an attempt to replicate the overall SiCE without the added factor of font ratio manipulation.

Method

Participants

Participants were 35 undergraduates (28 female, M_{age} = 19.91) from a large Midwestern university participating for course credit.

Stimuli & Procedure

The symbolic fraction pairs compared were the same set of 86 from Experiment 1. Font ratio magnitude was constant for all stimuli, as the numerator and denominator of each fraction were printed in the same font for a given fraction. To manipulate overall size, each comparison involved one fraction printed in a small size (74 x 230 pixels) and another in a large size (100 x 323 pixels). Thus, the screen area taken by small fractions was approximately half that taken by large ones. Absolute physical magnitude congruity was defined just as it was in Experiment 1. We manipulated absolute physical magnitude congruity (congruent, incongruent) and the location of the larger symbolic fraction (left, right) within subjects for each of the 86 fraction pairs. This yielded 344 trials for each participant (86 pairs x 2 absolute physical congruity x 2 side of presentation). All participants saw the same trials in random order. The procedure and computer hardware were identical to that of Experiment 1. Stimuli were presented using Superlab 5 software (Cedrus Corporation, 2014).

Results

We ran separate LMM analyses for RT and accuracy parallel to those conducted in Experiment 1. Independent variables included absolute physical magnitude congruity, holistic distance, numerator distance, denominator distance, and double symbolic incongruity, all defined the same as in Experiment 1. Responses more than 3 standard deviations from a participant's mean RT were trimmed. This affected less than 2% of the data. RT analyses included only error-free trials, but still included over 90% of all data points because participants were mostly accurate overall (M = 90.7). Results from LMM analyses are presented in Tables 2a and 2b and Fig. 5.

Insert Tables 2a and 2b about here

There was an SiCE for absolute physical magnitude, just as in Experiment 1. Participants were slower (M = 51.87 ms, $p < .01$) and less accurate (OR = .59, $p < .01$) when the overall physical sizes of a fraction pair were incongruent with the symbolic comparison dimension. There was also a double symbolic incongruity effect, as participants were slower (M = 422.16 ms, $p < .01$) and less accurate (OR = .17, $p < .01$) when the symbolic fraction with the smaller holistic value contained both a larger valued numerator and a larger valued denominator than the larger symbolic fraction.

Insert Fig. 5 about here

In addition to these congruity effects, participants were slower and less accurate when the distance between the holistic values of two fractions was smaller. As in Experiment 1, these effects were the largest effects we found. Participants were also slower and less accurate when numerator distances were smaller. Finally, although participants were slower when denominator distances were smaller, the componential distance effect for denominators regarding accuracy failed to reach significance. Again, the findings of both holistic and componentially based distance effects were consistent with findings from prior literature (DeWolf et al., 2014; Meert et al., 2010; Obersteiner et al., 2013).

Discussion

Experiment 2 confirmed our findings of an SiCE for fractions using a more direct paradigm than that of Experiment 1. Despite the fact that overall physical size was an irrelevant dimension, it still influenced judgments of symbolic magnitude. In order for participants to systematically map physical size onto symbolic size, they must differentiate between large and small fractions based on symbolic magnitude. Thus, these results are consistent with participants treating symbolic fractions as magnitudes that can be differentiated, not as generalized undifferentiated magnitudes less than one.

General Discussion

These experiments presented the first behavioral evidence that automatic processing of nonsymbolic magnitudes interferes with processing of symbolic fraction magnitudes. Experiment

1 revealed that even nonsymbolic ratios – defined not by the overall sizes of individual nonsymbolic components, but by the *relative* sizes of their component fonts – had significant effects on symbolic processing. Furthermore, both experiments demonstrated that an SiCE effect based on the summed area of fractions components is indeed operative during fraction comparison when stimuli are chosen that elicit holistic as opposed to componential processing of fractions values. These results provide new insights into the scope and nature of fraction magnitude processing.

Overall Size Congruity Effects

In contrast to Kallai and Tzelgov (2009), we found size congruency effects based on overall physical size in two separate experiments. We suspect the difference in findings was due to the fact that our choice of stimuli was more expansive and used fraction pairs without common components to induce holistic processing. By contrast, Kallai and Tzelgov's stimuli were all unit fractions, which multiple sources suggest lead to more componential processing. We interpret our findings to mean that processing fractions as holistic magnitudes does indeed lead to classic SiCEs, whereas componential processing does not. This parallels findings that eliciting holistic fractions processing also leads to classic distance effects whereas componential processing does not (Meert et al., 2010; Obersteiner et al., 2013; Schneider & Siegler, 2010).

It is critical to note an important difference between the current experiments and those of Kallai and Tzelgov (2009): Whereas Kallai and Tzelgov were primarily concerned with processing of symbolic fractions, we were concerned as much with the processing of nonsymbolic ratios as we were with symbolic fractions. Although nonsymbolic ratios are analogs of symbolic fractions, we do think of them as substantially different from symbolic fractions. Indeed, earlier work has suggested that adults compare nonsymbolic fraction magnitudes via a perceptual process that does not involve converting them to symbolic form (Matthews & Chesney, 2015). These differences should be kept in mind when comparing the current findings with those of Kallai and Tzelgov.

For instance, it is noteworthy that the SiCEs in the current experiments were accompanied by response latencies that averaged over 1600ms. Even though physical size clearly affected symbolic comparisons, it remains true that symbolic fractions comparisons took considerably longer than the < 800ms participants took to make fractions comparisons in Kallai and Tzelgov. This was longer still than the time those same participants typically took to compare whole numbers (typically ~500ms). The comparatively long reaction times might be taken to support Kallai and Tzelgov's contention that the magnitudes of symbolic fractions are not discretely represented in long-term memory and that they are instead generated by applying some sort of processing strategies to their whole number components (see also DeWolf et al., 2014). Thus, despite the findings of SiCEs, it remains clear that comparing symbolic fractions magnitudes is qualitatively more difficult than comparing whole numbers.

The Ratio Congruity Effect

Although Jacob and Nieder's (2009) neuroimaging paradigm found that nonsymbolic ratios automatically evoked neural responses, the current study is the first to show that automatic processing of nonsymbolic ratios can lead to competition with symbolic processing. This automatic processing of ratio magnitude even when irrelevant to the task at hand is consistent with Jacob et al.'s (2012) hypothesis that there are cognitive architectures specifically dedicated to processing nonsymbolically instantiated ratios. These results stand alongside developmental work on ratio processing (e.g., Boyer & Levine, 2012; Duffy et al., 2005; Sophian, 2000) in contrast to the assertion made by several cognitive scientists that fractions concepts are unsupported by primitive architectures (e.g., Dehaene, 1997; Feigenson et al., 2004; Gelman & Williams, 1998).

The presence of RCEs is evidence of considerably more complex nonsymbolic processing than that indicated by SiCEs for overall size. Whereas overall SiCEs may reflect automatic processing of coarse scalar magnitudes that correspond to the overall amount of space stimuli occupy, RCEs persisted even after partialing out absolute physical magnitude, confirming that it really was the ratio between font areas that drove the effect. This is evidence of automatic processing of relational magnitudes, minimally with genuine ordinal properties: For congruity to exert effects, the direction of 'large' in the symbolic dimension must be at least coarsely mapped to 'large' in the nonsymbolic ratio dimension. This indicates a degree of sophistication in nonsymbolic ratio processing that goes beyond a generalized representation of small magnitudes.

One question raised by the presence of RCEs is whether the same neural circuits are involved in the processing of both nonsymbolic ratios and symbolic fraction values. Indeed, neuroimaging studies have shown that the fronto-parietal cortical networks implicated in the representation of nonsymbolic ratios are similar to those involved in representing and processing symbolic fractions (Jacob & Nieder, 2009). However, no neuroimaging studies have directly investigated whether processing of symbolic fractions and of nonsymbolic ratios engage the same neural circuitry. The present findings suggest that the time for investigating these links has arrived.

Conclusions

On a final note, the revelation of RCEs raises questions concerning the nature of human numerical cognition as it results to magnitude processing more generally. Is the automatic processing of nonsymbolic ratio magnitude the activity of a very general magnitude processing system (Walsh, 2003)? Might this automatic ratio processing help support our understanding of numerical magnitudes (Lewis et al., 2015; Matthews, Lewis, & Hubbard, 2015)? Indeed, Siegler et al. (2011) argued that the one thing uniting whole numbers, fractions, and all real numbers for that matter, is that they can be represented as magnitudes on a number line. It may be that number lines are such an effective representation because they leverage the same powerful and automatically invoked sensitivities to nonsymbolic ratios that drove the effects of this research (See Barth & Paladino, 2011 for an account of number line estimation as ratio matching). Moreover, the current research stands alongside recent findings demonstrating that ratio sensitivity extends beyond line segments minimally to include ratios composed of dots (Matthews et al., 2015; Meert, Grégoire, Seron, & Noël, 2012), circle areas (Matthews & Chesney, 2015), and implicitly defined font area ratios. In the final analysis, it may be that attending more to how humans process nonsymbolic ratios, these fractions that we can't ignore, may hold significant potential for enriching our understanding of the human number sense.

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Appendix: Symbolic Fraction Pairs

$^{1}/_{3}$_$^{4}/_{9}$, $^{2}/_{9}$_$^{1}/_{3}$, $^{2}/_{3}$_$^{7}/_{9}$, $^{5}/_{9}$_$^{2}/_{3}$, $^{1}/_{4}$_$^{3}/_{8}$, $^{3}/_{4}$_$^{7}/_{8}$, $^{5}/_{8}$_$^{3}/_{4}$, $^{3}/_{7}$_$^{5}/_{9}$, $^{2}/_{3}$_$^{4}/_{5}$, $^{3}/_{4}$_$^{8}/_{9}$, $^{1}/_{4}$_$^{2}/_{5}$, $^{2}/_{9}$_$^{3}/_{8}$, $^{5}/_{8}$_$^{7}/_{9}$, $^{4}/_{9}$_$^{3/}{}_{5}$, $^{2}/_{7}$_$^{4}/_{9}$, $^{1}/_{8}$_$^{2}/_{7}$, $^{2}/_{5}$_$^{4}/_{7}$, $^{1}/_{9}$_$^{2}/_{7}$, $^{5}/_{7}$_$^{8}/_{9}$, $^{1}/_{5}$_$^{3}/_{8}$, $^{3}/_{5}$_$^{7}/_{9}$, $^{1}/_{4}$_$^{3}/_{7}$, $^{3}/_{8}$_$^{5}/_{9}$, $^{4}/_{9}$_$^{5}/_{8}$, $^{5}/_{9}$_$^{3}/_{4}$, $^{3}/_{8}$_$^{4}/_{7}$, $^{3}/_{7}$_$^{5}/_{8}$, $^{2}/_{9}$_$^{3}/_{7}$, $^{2}/_{3}$_$^{7}/_{8}$, $^{1}/_{3}$_$^{5}/_{9}$, $^{4}/_{9}$_$^{2}/_{3}$, $^{2}/_{3}$_$^{8}/_{9}$, $^{1}/_{5}$_$^{3}/_{7}$, $^{1}/_{7}$_$^{3}/_{8}$, $^{1}/_{3}$_$^{4}/_{7}$, $^{1}/_{5}$_$^{4}/_{9}$, $^{1}/_{7}$_$^{2}/_{5}$, $^{1}/_{9}$_$^{3}/_{8}$, $^{4}/_{9}$_$^{5}/_{7}$, $^{2}/_{7}$_$^{5}/_{9}$, $^{1}/_{8}$_$^{2}/_{5}$, $^{3}/_{5}$_$^{7}/_{8}$, $^{3}/_{5}$_$^{8}/_{9}$, $^{1}/_{9}$_$^{2}/_{5}$, $^{1}/_{3}$_$^{5}/_{8}$, $^{1}/_{7}$_$^{4}/_{9}$, $^{1}/_{8}$_$^{3}/_{7}$, $^{1}/_{4}$_$^{5}/_{9}$, $^{2}/_{7}$_$^{3}/_{5}$, $^{1}/_{9}$_$^{3}/_{7}$, $^{4}/_{7}$_$^{8}/_{9}$, $^{5}/_{9}$_$^{7}/_{8}$, $^{1}/_{8}$_$^{4}/_{9}$, $^{2}/_{7}$_$^{5}/_{8}$, $^{3}/_{8}$_$^{5}/_{7}$, $^{2}/_{9}$_$^{4}/_{7}$, $^{1}/_{4}$_$^{3}/_{5}$, $^{2}/_{5}$_$^{3}/_{4}$, $^{1}/_{5}$_$^{4}/_{7}$, $^{3}/_{7}$_$^{4}/_{5}$, $^{1}/_{4}$_$^{5}/_{8}$, $^{2}/_{9}$_$^{3}/_{5}$, $^{2}/_{5}$_$^{7}/_{9}$, $^{1}/_{3}$_$^{5}/_{7}$, $^{2}/_{9}$_$^{5}/_{8}$, $^{3}/_{8}$_$^{7}/_{9}$, $^{1}/_{7}$_$^{5}/_{9}$, $^{1}/_{4}$_$^{2}/_{3}$, $^{3}/_{8}$_$^{4}/_{5}$, $^{1}/_{8}$_$^{5}/_{9}$, $^{4}/_{9}$_$^{7}/_{8}$, $^{1}/_{3}$_$^{7}/_{9}$, $^{1}/_{8}$_$^{4}/_{7}$, $^{1}/_{7}$_$^{3}/_{5}$, $^{1}/_{9}$_$^{4}/_{7}$, $^{3}/_{7}$_$^{8}/_{9}$, $^{2}/_{7}$_$^{3}/_{4}$, $^{1}/_{4}$_$^{5}/_{7}$, $^{1}/_{3}$_$^{4}/_{5}$, $^{1}/_{5}$_$^{2}/_{3}$, $^{1}/_{8}$_$^{3}/_{5}$, $^{2}/_{5}$_$^{7}/_{8}$, $^{1}/_{7}$_$^{5}/_{8}$, $^{1}/_{9}$_$^{3}/_{5}$, $^{2}/_{5}$_$^{8}/_{9}$, $^{2}/_{9}$_$^{5}/_{7}$

Figure 1. (a) Sample nonsymbolic ratio discrimination task used with monkeys that required matching ratios composed of pairs of line lengths. Monkeys were nearly as accurate as well educated human adults (85.6% vs. 92%). Figure reproduced from Jacob, Vallentin, and Nieder (2012). (b) Sample of cross-format comparison task from Matthews and Chesney (2015). Participants completed nonsymbolic ratio comparisons across tasks more quickly than they completed symbolic comparisons.

Figure 2. Non-symbolic dimensions manipulated. Left: Absolute physical magnitude was defined by the summed areas taken up by the implicit bounding boxes around the numerator (light gray) and denominator (dark gray) for a given fraction. This was defined independently of nonsymbolic font ratio. Right: Nonsymbolic font ratio was defined by the ratio of the area of the implicit bounding box around the numerator to that of the bounding box around the denominator. This was independent of the absolute physical magnitude. Variables stand in place of numbers in the figure to illustrate that these dimensions vary independently of the symbolic numerical ratios presented. The figure depicts bounding boxes as shaded for the purpose of clarity. Actual stimuli were presented without bounding boxes as shown in Fig. 3.

Figure 3. Example stimuli as presented in Experiment 1. Fraction pairs were presented in all four combinations of font and absolute magnitude congruity.

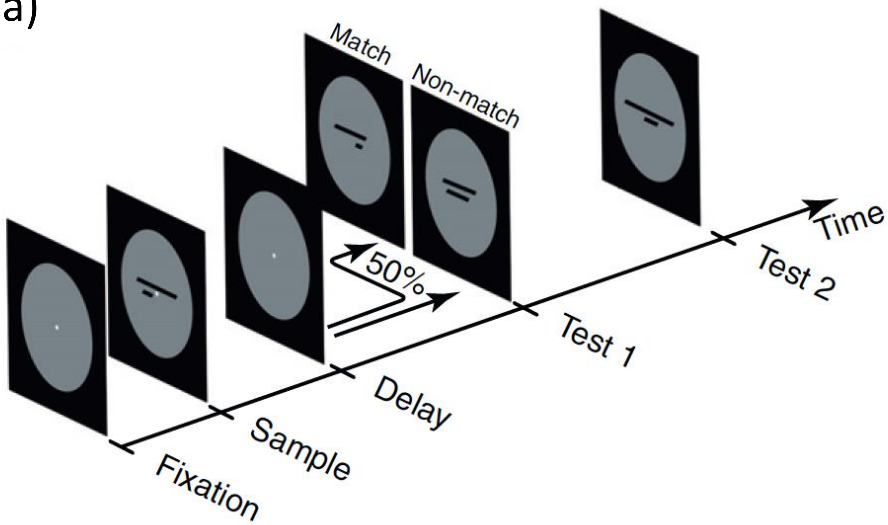
Figure 4. Mean RT (top) and accuracy (bottom) as a function of font ratio and absolute physical magnitude congruity and symbolic fraction distance. Participants responded more slowly and less accurately when font ratio magnitude was incongruent with symbolic fraction decisions

(indicated by differences between solid and dotted lines) and as the holistic distance between the two symbolic fractions decreased (as indicated by the slopes of all lines). Participants were also slower and less accurate when the absolute physical magnitude (overall size) was incongruent with the symbolic decision (indicated by the differences between panels on the left and on the right). Note: Although all individual data points were entered for regressions, points were aggregated into distance bins for the figure. For instance (.1, .2) on the x-axis includes all comparisons of distance between .1 and .2.

Figure 5. Mean RT (left) and accuracy (right) as a function of absolute physical magnitude congruity and symbolic fraction distance. Participants were slower and less accurate when the absolute physical magnitude (overall size) was incongruent with the symbolic decision (indicated by the differences between dotted and solid lines). Participants also responded more slowly and less accurately when the holistic distance between the two symbolic fractions decreased (as indicated by the slopes of all lines). Note: Although all individual data points were entered for regressions, points were aggregated into distance bins for the figure. For instance (.1, .2) on the x-axis includes all comparisons of distance between .1 and .2.

Fig_1

a)


Match
Non-match
50%
Time
Test 2
Test 1
Delay
Sample
Fixation

b)

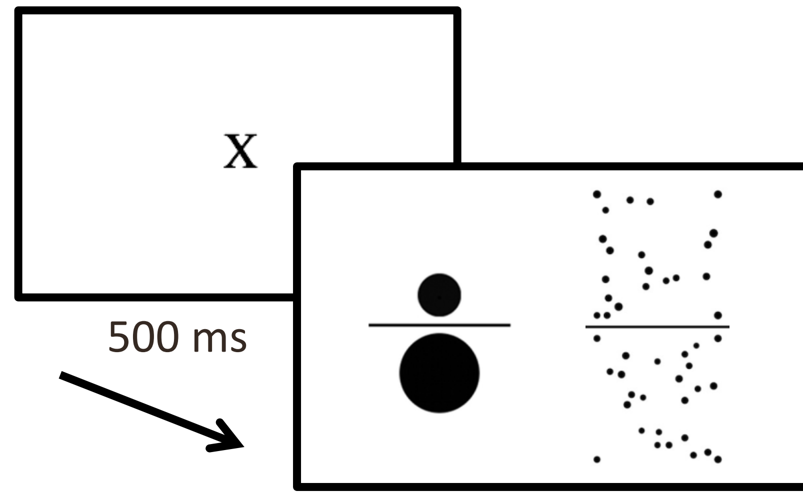
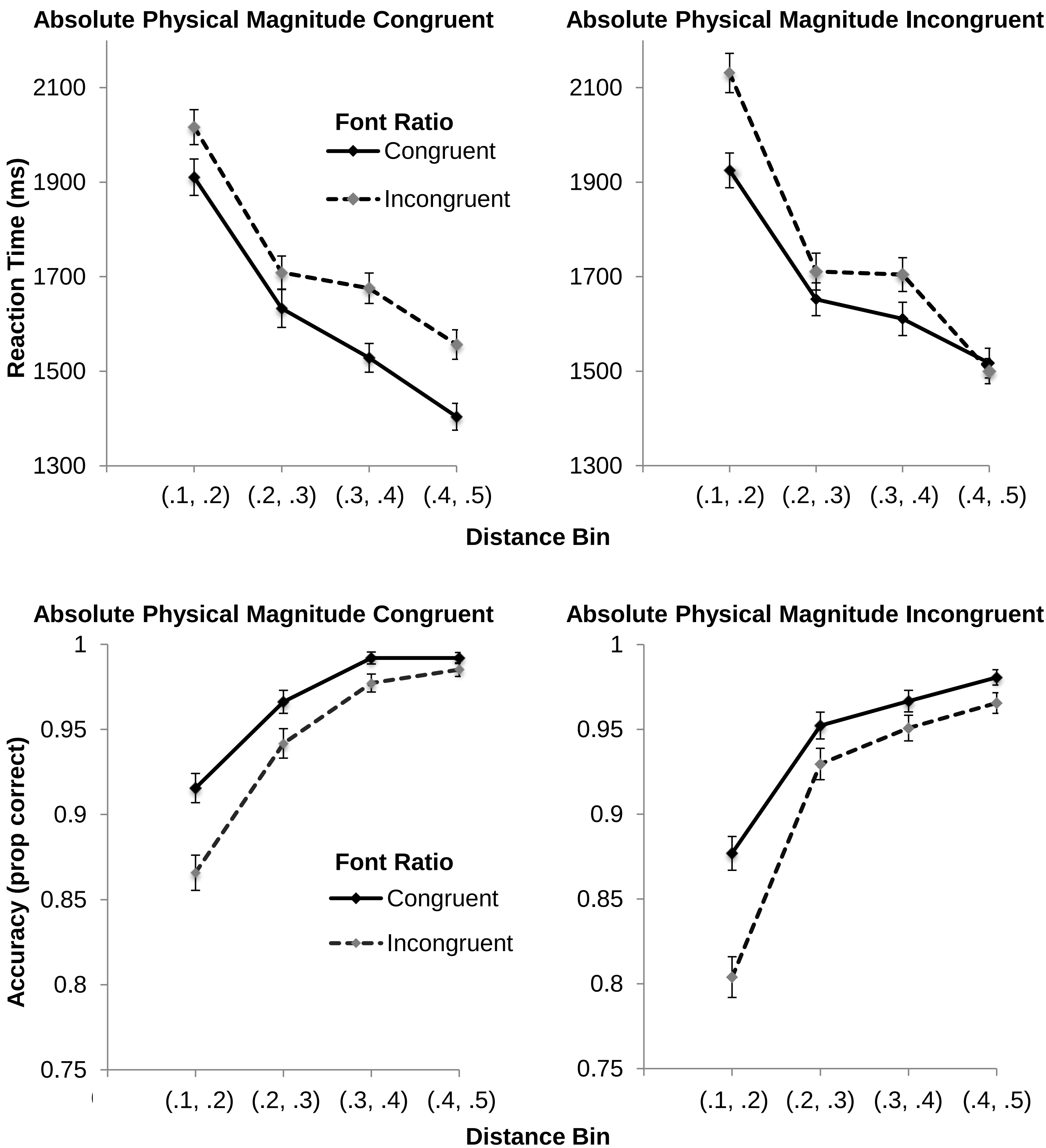

X
500 ms

Fig_4



Absolute Physical Magnitude Congruent
Absolute Physical Magnitude Incongruent
Reaction Time (ms)
2100
1900
1700
1500
1300
Font Ratio
Congruent
Incongruent
(.1, .2)
(.2, .3)
(.3, .4)
(.4, .5)
Distance Bin
Absolute Physical Magnitude Congruent
Absolute Physical Magnitude Incongruent
Accuracy (prop correct)
1
0.95
0.9
0.85
0.8
0.75
Font Ratio
Congruent
Incongruent
Distance Bin

Table 1a.

Linear MLM Regression Results for Response Time, Experiment 1.

	Response Time (Linear Mixed Model)		
	Mean slope	[95% CI]	*p*
Font ratio congruity (incongruent = 1)	99.35	[65.20, 133.50]	<.01
Absolute magnitude congruity (incongruent = 1)	35.50	[1.35, 69.64]	.04
Holistic distance	-1220.20	[-1378.49, -1061.91]	<.01
Numerator distance	-46.96	[-62.12 -31.80]	<.01
Denominator distance	-34.38	[-48.34, -20.12]	<.01
Double symbolic (incongruent = 1)	507.67	[47.50, 600.78]	<.01

Note: For response time, a positive mean slope indicates that increasing values of a factor resulted in a slower response time.

Table 1b.

Logistic MLM Regression Results for Accuracy, Experiment 1.

	Accuracy (Logistic Mixed Model)			
	Mean β	[95% CI]	*Odds Ratio*	*p*
Font ratio congruity (incongruent = 1)	-.62	[-.78, -.47]	.54	<.01
Absolute magnitude congruity (incongruent = 1)	-.57	[-.73, -.42]	.57	<.01
Holistic distance	5.58	[4.71, 6.45]	265.07	<.01
Numerator distance	.04	[-.05, .12]	1.04	.41
Denominator distance	-.06	[-.13, .00]	.94	.08
Double symbolic (incongruent = 1)	-1.93	[-2.21, -1.66]	.15	<.01

Note: For accuracy, a negative mean β indicates that a factor rendered participants less likely to be correct compared to baseline, as reflected by odds ratios of less than 1.

Table 2a.

Linear MLM Regression Results for Response Time, Experiment 2.

	Response Time (Linear Mixed Model)		
	Mean slope	[95% CI]	*p*
Absolute magnitude congruity (incongruent = 1)	51.87	[20.70, 83.04]	<.01
Holistic distance	-730.35	[-875.48, -585.23]	<.01
Numerator distance	-42.80	[-56.68, -28.92]	<.01
Denominator distance	-10.38	[-23.39, 2.63]	.12
Double symbolic (incongruent = 1)	422.16	[335.01, 509.31]	<.01

Note: For response time, a positive mean slope indicates that increasing values of a factor resulted in a slower response time.

Table 2b.

Logistic MLM Regression Results for Accuracy, Experiment 2.

	Accuracy (Logistic Mixed Model)			
	Mean β	[95% CI]	*Odds Ratio*	*p*
Absolute magnitude congruity (incongruent = 1)	-.53	[-.67, -.39]	.59	<.01
Holistic distance	3.52	[2.82, 4.23]	33.78	<.01
Numerator distance	.12	[.06, .19]	1.13	<.01
Denominator distance	-.06	[-.12, .00]	.94	.07
Double symbolic (incongruent = 1)	-1.79	[-2.05, -1.54]	.17	<.01

Note: For accuracy, a negative mean β indicates that a factor rendered participants less likely to be correct compared to baseline, as reflected by odds ratios of less than 1.

Fig_2

Absolute Physical Magnitude

Nonsymbolic Font Ratio

Ratio R_1 (Large) | Ratio R_2 (Small) | Ratio R_2 (Large) | Ratio R_1 (Small)

Area X_1 = 46 * 64 = 2944
Area Y_1 = 70 * 98 = 6860

Area X_2 = 41 * 57= 2337
Area Y_2 44 * 62 = 2728

Area X_2 = 41 * 57= 2337
Area Y_2 44 * 62 = 2728

Area X_1 = 46 * 64 = 2944
Area Y_1 = 70 * 98 = 6860

Absolute Magnitude R_1
Area X_1 + Area Y_1 = 9804

Absolute Magnitude R_2
Area X_2 + Area Y_2 = 5065

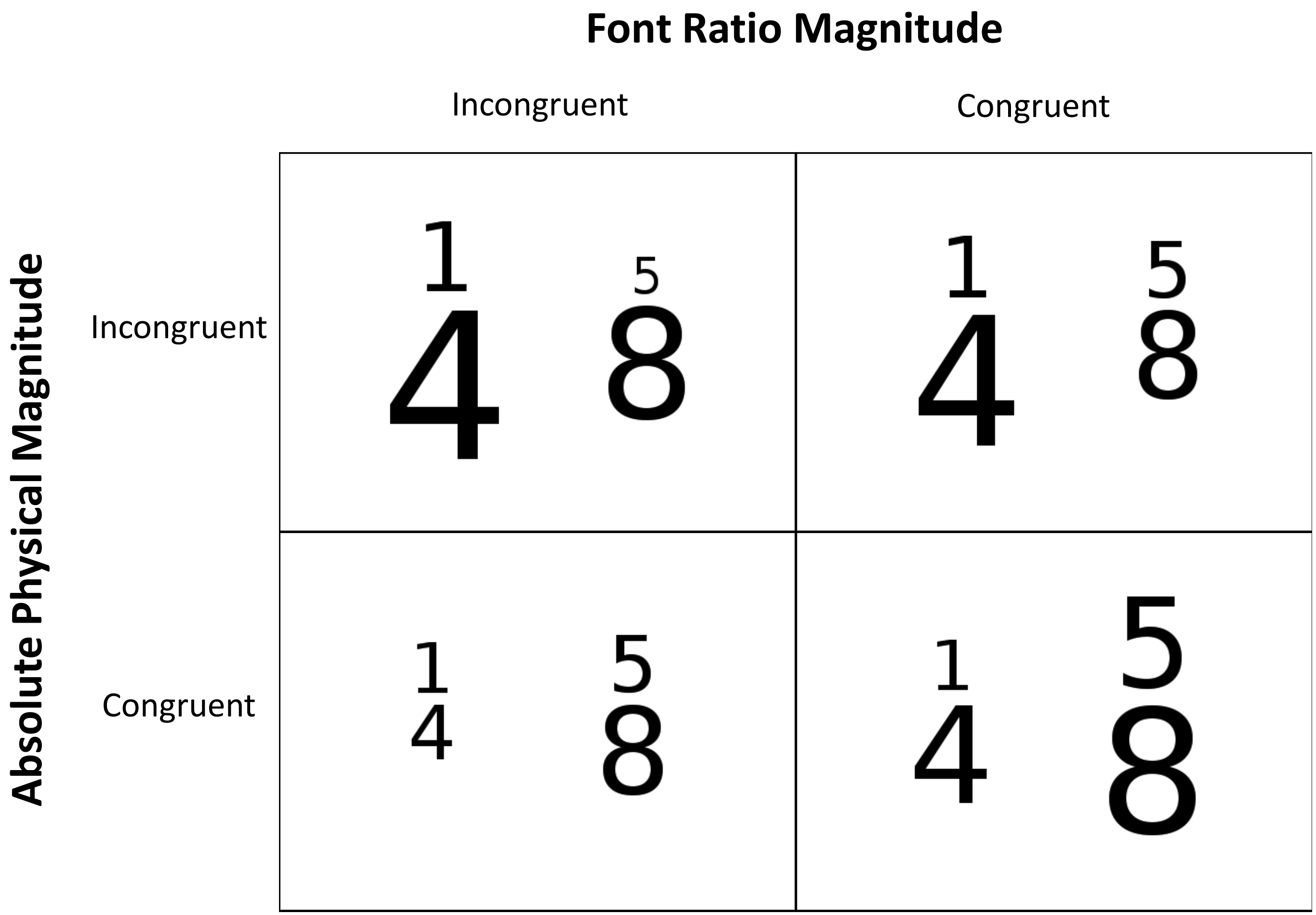
Font Ratio R_1
Area X_1 ÷ Area Y_1 = .86

Font Ratio R_2
Area X_2 ÷ Area Y_2 = .43

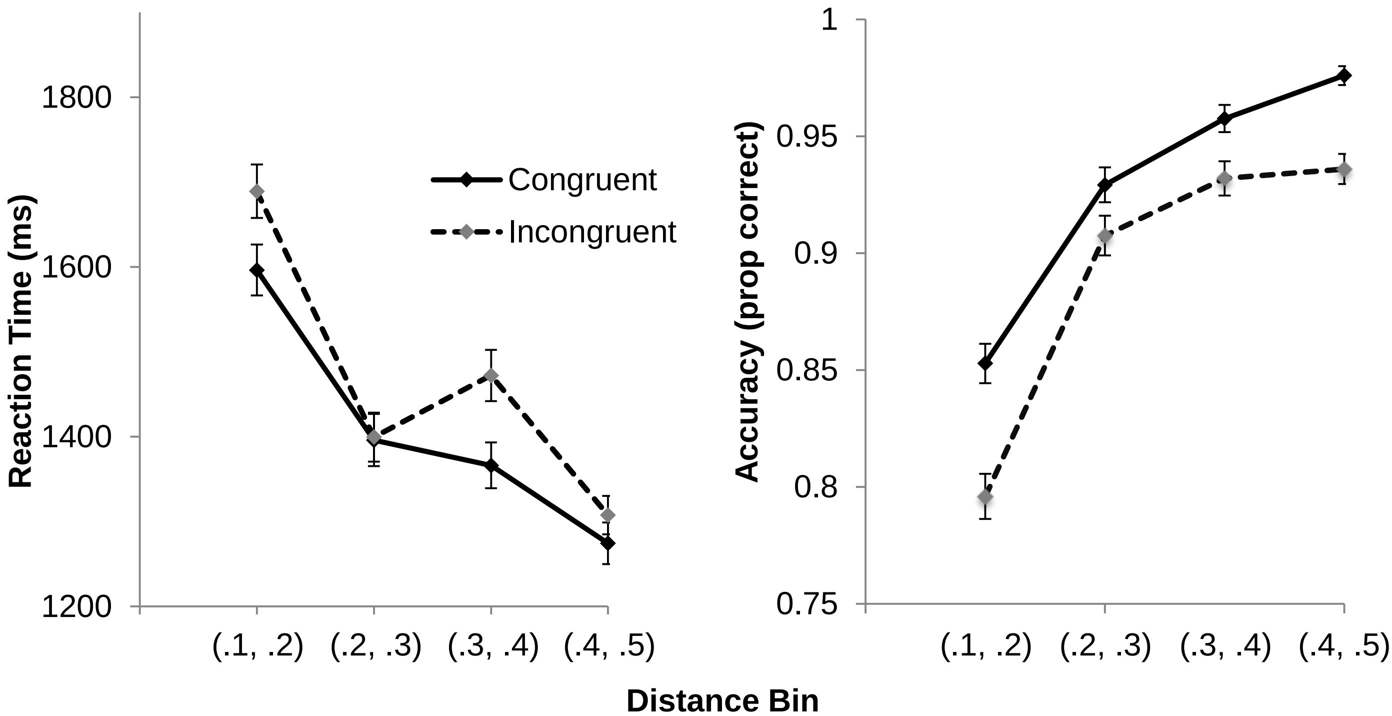
Absolute Magnitude R_1 : Absolute Magnitude R_2 – 9804/5065 ≈ 2:1

Font Ratio R_2 : Font Ratio R_1 – .86:.43 = 2:1

Figure


Font Ratio Magnitude
Incongruent
Congruent
Absolute Physical Magnitude
Incongruent
Congruent
1
4
5
8
1
4
5
8
1
4
5
8
1
4
5
8

Figure



Reaction Time (ms)
1800
1600
1400
1200
Congruent
Incongruent
(.1, .2)
(.2, .3)
(.3, .4)
(.4, .5)
Accuracy (prop correct)
1
0.95
0.9
0.85
0.8
0.75
(.1, .2)
(.2, .3)
(.3, .4)
(.4, .5)
Distance Bin